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BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH132 -first EXAM-
SUMMER COURSE 2017/2018

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Question 1. *(30 points)* Circle the correct answer.

1. the Taylor series at $x = 0$ of the function $f(x) = \frac{1}{1-2x}$ is :

$r < 1$

$2x < 1$

$x < \frac{1}{2}$

a) $\sum_{n=0}^{\infty}(2x)^n, |x| < 1$

b) $\sum_{n=0}^{\infty}(2x)^n, |x| < \frac{1}{2}$

c) $\sum_{n=0}^{\infty}(-1)^n(2x)^n, |x| < 1$

2. The series $\sum_{n=1}^{\infty}\frac{2^{-n}+n}{n^3+n^2}$.

$\frac{2+n}{n^3+n^2} < \frac{1}{n^2} = \frac{n}{n^3}$

a) Diverges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n^3}$

$n^2 < n^3+n^2$

b) Converges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$\frac{1}{n^2}$

c) Converges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{2^{-n}}$

d) Diverges by limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n}$

Root
ratio

$r<1$
Convey
$r>1$. di

$\left(\frac{e^{2n}}{n^n}\right)^{\frac{1}{n}} = \left(\frac{e^2}{n}\right) = 0 \Rightarrow \underline{0<1}$

7 3. The series $\sum_{n=1}^{\infty}\frac{e^{2n}}{n^n}$

$\frac{e^0}{\infty} \cdot \frac{2e^{2n}}{n^2}$

a) Diverges by the nth term test.

xb) Diverges by the integral test.

c) Converges by the root test.

$\frac{e^2}{n} = \frac{e}{\infty} = \underline{0<1}$ Converg by root

d) Diverges by the root test.

$\lim_{n\to\infty}\frac{e^{2n}}{n^n} = \frac{\infty}{\infty}$ | $\lim \frac{e^2}{n}$

$\frac{e^2}{n} = \frac{e^2}{\infty} = 0 < 1$

$\frac{2e^{2n}}{n^2}$

4. The values of x for which we can replace $\frac{1}{1+x}$ by $1-x+x^2$ with an error of magnitude less than 64×10^{-6} are

$\frac{1}{1+x} = 1 - x + x^2 - x^3$

error $< (64 \times 10^{-6})^3$

$x^3 < 64 \times 10^{-6}$

$x^3 = \sqrt{64 \times 10^{-6}}$

4×10^{-3}

a) $|x| < 4 \times 10^{-2}$

b) $|x| < 8 \times 10^{-3}$

c) $|x| < 64 \times 10^{-6}$

d) None of the above.

5. The series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$u = \ln n$

$du = \frac{dx}{x}$

$f(x) = \frac{1}{xu}$

$\int \frac{1}{u} = \ln u \Big|_{\ln 2}^{\infty}$

a) Converges by the root test.

b) Diverges by the root test.

c) Converges by the integral test.

d) Diverges by the integral test.

6. $1 + \ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \ldots.. =$

$e^{\ln 3}$ e^x but $x = \ln 3$ $e^{\ln 3}$

$(x, -\infty)$ $(\infty, -\infty)$

a) e^3 .

b) e .

c) 3 .

d) e - 1 .

7. The binomial series of $\sqrt{1+x^3}$ is

$(1+x^3)^{\frac{1}{2}}$

$1 + \binom{m}{k} x^k$

$1 + \binom{\frac{1}{2}}{1} x^3 + \binom{\frac{1}{2}}{2} x^6 + \binom{\frac{1}{2}}{3} x^9$

$1 + \frac{x}{2} + \frac{\frac{1}{2} \cdot \frac{-1}{2}}{2!} x^2 + \frac{\left(\frac{1}{2} \cdot \frac{-1}{2} \cdot \frac{-3}{2}\right)}{3!} x^3$

$1 + \frac{x}{2} - \frac{x^2}{8} + \frac{-3}{8 \cdot 3!} x^3$

$-\frac{x^3}{16}$

$\frac{-1}{2}$ $\frac{-2}{2}$ $\frac{-3}{2}$

a) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots$

b) $1 + \frac{x^3}{2} - \frac{x^6}{8} + \frac{3x^9}{16} - \cdots$

c) $1 + \frac{x^3}{2} - \frac{x^6}{8} + \frac{x^9}{16} - \cdots$

مفتوح $(\, , \,)$ $(\infty, -\infty)$

مغلق $[\, , \,]$

8. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ is:

R $\frac{x^n}{n^{\frac{1}{2}}}$

$\frac{|x|}{n}$

$\frac{-3}{8 \cdot 3 \cdot 2 \cdot 1}$

$\frac{x^n}{n^{\frac{1}{2}}}$ $\frac{x^{n+1}}{(n+1)^{\frac{1}{2}}} \cdot \frac{n^{\frac{1}{2}}}{x}$

a) [-1,1].

b) (-1,1)

$-1 < x < 1$

c) [-1,1)

d) (-1,1]

$x = 1$ $\frac{1}{n^{\frac{1}{2}}}$ div hormonin

$|x| \frac{n^{\frac{1}{2}}}{n+1}$

$|x|$ $-1 < x < 1$

P-series $1 \geq P$ div

P-series $1 < P$ conv

$\frac{-1}{n^{\frac{1}{2}}}$

conve condionall

$\frac{2}{2} - \frac{1}{2}$

9. $\sum_{n=0}^{\infty}\left(\frac{1}{2^n} + \frac{(-1)^n}{3^n}\right)$ $\quad \sum_{n=0} \left(\frac{1}{2}\right)^n + \left(-\frac{1}{3}\right)^n$ $\quad a=1 \quad a=1$

a) $\frac{5}{4}$ $\quad \frac{1}{1-\frac{1}{2}} + \frac{1}{1+\frac{1}{3}}$ $\quad \frac{3}{3} + \frac{1}{3}$ $\quad \frac{4}{3}$

b) $\frac{11}{4}$ $\quad \frac{4\times 2}{4\times 1} + \frac{3}{4}$

c) $\frac{7}{2}$ $\quad \frac{8+3}{4}$

d) none of the above

10. If we approximate e^x by $1 + x + \frac{x^2}{2}$, then the error of estimating when $|x| < 0.1$ less than

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ $\quad R$

a) $\frac{e^{0.1}}{8000}$ $\quad \frac{e^1 (0.1)^3}{6} \quad \frac{e^{0.1}}{6000}$ $\quad 3\cdot 2$ $\quad 6$

b) $\frac{e^{0.1}}{6000}$

c) $\frac{(0.1)^3}{3!}$

11. The sum of the telescoping series $\sum_{n=1}^{\infty} \frac{3}{n^2} - \frac{3}{(n+1)^2} =$

Sum $= \left(3 - \frac{3}{(2)^2}\right) + \left(\frac{3}{(2)^2} - \frac{3}{3^2}\right) + \left(\frac{3}{3^2} - \frac{3}{4^2}\right) + \left(\frac{3}{(n-1)^2} - \frac{3}{n^2}\right) + \left(\frac{3}{n^2} - \frac{3}{(n+1)^2}\right)$

a) 0 $\quad 3 - \frac{3}{(n+1)^2} = 3$

b) 3

c) 1

d) This series diverges.

12. The radius of convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$ is

by root: $\left(\left(\frac{n}{n+1}\right)^{n^2} x^n\right)^{\frac{1}{n}}$

a) 1 $\quad \left(\frac{n}{n+1}\right)^n \quad (x^{\frac{n}{n}})$ $\quad \left(\frac{n}{n+1}\right) = \frac{1}{1+\frac{1}{n}} = \frac{1}{e}$

b) e

c) e^{-1}

d) ∞

$\frac{|x|}{e} \Rightarrow -e < x < e$

radius $= \frac{e+e}{2} = \frac{2e}{2} = e$

13. The sequence $\{a_n\} = \left(3 + \frac{3}{n}\right)^n$ is : $\quad \left(1 + \frac{x}{n}\right)^n = e^x$

a) diverge . $\quad \left(3 + \frac{3}{n}\right)^n = e^3$

b) converge to e^3.

c) converge to e.

d) converge to e^{-3}.

14. The series $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

a) Converges by the ratio test.

b) Diverges by the ratio test.

c) Diverges by the nth term test.

d) Converges by the nth term test.

$\frac{(n+1)\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{n \ln n} = \frac{n+1}{n} \cdot \frac{\ln(n+1)}{\ln n} \cdot \frac{2^n}{2^{n+1}}$

$1 \cdot 1 \cdot \frac{1}{2}$

15. the partial sum of the series $\sum_{n=2}^{\infty} \frac{-2}{n(n+1)}$

a) $S_n = \frac{1}{2} - \frac{2}{n+1}$

b) $S_n = \frac{1}{2} + \frac{2}{n+1}$

c) $S_n = -2 + \frac{2}{n+1}$

d) $S_n = -1 + \frac{2}{n+1}$

$= \frac{A}{n} + \frac{B}{n+1} = -2$

$A(n+1) + B(n) = -2$

$n=0 \quad A = -2$

$n=-1 \quad \boxed{B = 2}$

16. The series $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n+1}$

a) converge conditioally

b) diverge

c) converge geometric series

d) converge absolutley

$\frac{2^n}{3^n+1} = a_n$

$\lim \frac{a_n}{b_n} = \frac{2^n}{3^n+1} \cdot \frac{3^n}{2^n} = \frac{6^n}{6^n} = 1$ both conv or both div

b_n conv

17. The series $\sum_{n=1}^{\infty} \frac{e^n}{e^n+1} = 1$

a) converge to 1

b) converge by integral test

c) diverges by nth term test

L.M.T $\frac{a_n}{b_n} = L$ both conv or both div

18. The Maclaurian series of $f(x) = \ln(1+x^2)$ is

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{2n}}{n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n!}$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n}$

d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^2}{n}$

$\ln x = \sum_{n=1} (-1)^{n-1} \frac{x^n}{n}$ or $\sum_{n=0} (-1)^n \frac{x^{n+1}}{n+1}$

$(-1)^{n-1} \frac{x^{2n}}{n}$

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19. The series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ $= a_n$

b_n $\frac{n^{\frac{1}{2}}}{n^2} = \frac{1}{n^2 \cdot n^{\frac{1}{2}}} = \frac{1}{n^{3/2}} =$ Conv p-series

$\lim \frac{a_n}{b_n} = \frac{\sqrt{n}}{n^2+1} \cdot n^{3/2}$

$n^{\frac{1}{2}}$

$\frac{1}{n^{3/2}}$

a) diverges by the ratio test

b) Converges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

c) diverge by the nth term test

d) diverge by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

20. The alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^2+5}$

$n^2+1 > n^{7/2}$

$\frac{\sqrt{n}}{n^2+1} < \frac{1}{n^{7/2}}$

a) Converges absolutely.

b) diverge by alternating series test

c) Converges conditionally

d) diverges by nth term test

$\lim_{n\to\infty} u_n \frac{n^2+1}{n^2+5} = 1 \neq 0$ div by nth tes

$\frac{n}{n^2} = \frac{1}{n}$

Question 2. *(7 points)* determine if the following series converge absolutely, conditionally or diverge

$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$ — positive, decreasing, $\lim_{n\to\infty} u_n = 0$

$\lim_{n\to\infty} |u_n| = \frac{1+n}{n^2} = a_n$

$b_n = \sum_{n=0} \frac{1}{n}$ div hormine seric

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1+n}{n^2}, \; n = \frac{n+n^2}{n^2} = \frac{n^2}{n^2} = 1 \Rightarrow$ both conv or both di

but bn diverge $\Rightarrow b_n = \sum \frac{1}{n}$ hormonic or p-series } diverges

since $\sum(-1)^{n+1}\frac{1+n}{n^2}$ converge conditonally

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Question 3. *(5 points)*
use differentiation for $\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$, to find the Maclaurin series of $f(x) = \frac{1}{(1+x)^2}$

$\ln(1+x) = x + \frac{x^2}{2} + \frac{x^3}{3} + x^4 \quad \sum_{n=0}^{\infty} x^n$

$\ln(1+x^2) = \sum_{n=2}^{\infty}(n-1)\,n\,x^{n-2}$

$\ln(1+x^2) = \sum_{n=0}^{\infty}(n+1)(n+2)\,x^n$

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{(1+x)}, \quad f''(x) = \frac{1}{(1+x)^2}$

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$$\frac{a_{n+1}}{a_n} = \frac{(x+2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n\,2^n}{(x+2)^n}$$

$|x+2|$

Question 4. *(13 points)* determine the radius and the interval of convergence of the series for what values of x does the series converges conditionally ?

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+2)^n}{n2^n}$$ ~~by root~~ $\Rightarrow$ ~~$\lim_{n\to\infty} x+2$~~

by ratio

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{(x+2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{(x+2)^n} \Downarrow$$

$$\Rightarrow |x+2| \lim_{n\to\infty}\frac{n\,2^n}{n+1\;2^{n+1}} = \left|\frac{x+2}{2}\right| < 1$$

$$\Rightarrow -1 < \frac{x+2}{2} < 1 \Rightarrow -2 < x+2 < 2 \Rightarrow \boxed{-4 < x < 0}$$

$$\text{Radius} = \frac{0+4}{2} = \frac{4}{2} = 4$$

$$\text{Center} = \frac{0+-4}{2} = \frac{-4}{2} = -2$$

$$x=0 \quad \sum_{n=0}^{\infty}\frac{2^n}{n2^n} \Rightarrow \sum_{n=0}^{\infty}\frac{1}{n} \quad \rightarrow \text{div by p-series or hormonic series}$$

$$x=-4, \sum(-1)^{n+1}\frac{(-4+2)^n}{n2^n} \Rightarrow \sum_{n=0}^{\infty}\frac{(-1)^{n+1}(-2)^n}{n2^n} \Rightarrow \sum_{n=0}^{\infty}(-1)^{2n+1}\frac{2^n}{n2^n}$$

positive

decreasing

$\lim_{n\to\infty} u_n = 0$

$$\sum(-1)^{2n+1}\frac{2^n}{n2^n} = \sum_{n=0}^{\infty}(-1)^{2n+1}\frac{1}{n} \Rightarrow \text{converge Conditionally}$$

Interval of converge $-4 \leq x < 0$

Converge Condronaly at $x = -4$

diverges at $x \geq 0$ or $x < -4$

one houres

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